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Non-abelian Born-Infeld action and solitons for critical non-BPS branes

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Abstract: The non-abelian flat directions in the tachyon potential of stable non-BPS branes recently found are shown to persist to all orders in α' at tree level in the string theory. We also obtain the non-abelian Born-Infeld action including the tachyon potential for a stack of stable non-BPS branes on a critical orbifold. Finally we discuss stable soliton states on the non-BPS brane.

Keywords: Solitons Monopoles and Instantons, D-branes, Conformal Field Models in String Theory.

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1. Introduction

D-branes play an important role in understanding non-perturbative aspects of string theory and supersymmetric field theory alike [1]. While the dynamics of D-branes are encoded in the open strings ending on them, the low energy effective action, to leading order in α', has been shown to be that of dimensionally reduced, ten-dimensional supersymmetric Yang-Mills theory [2]. For constant field strength and as long as the second derivatives of the scalars in the Yang-Mills mutiplet are small, higher order corrections in α' are described in terms of the Born-Infeld (BI) action [3]–[10].

More recently it has become clear that in addition to D-branes the spectrum of type-II string theory contains non-supersymmetric but, nevertheless, stable states [11]–[16]. At leading order the dynamics of these non-BPS D-branes is again described by a field theory. However, in contrast to D-branes, this field theory is not obtained from dimensionally reduced ten-dimensional Yang-Mills theory. In particular, there are extra scalars which originate in a tachyonic sector of the open strings ending on these branes. To leading order in α' the field theory of a stack of stable non-BPS D-branes was obtained in [17]. A, maybe unexpected, feature of this theory is that at certain points in the moduli space of stable non-BPS D-branes the potential for the "tachyons"[1] has non-abelian flat directions. An immediate question is, of course, whether or not these flat directions, which are not protected by supersymmetry, are removed by higher order corrections. On general grounds, we expect that α' corrections to the field theory approximation should again be described by some Born-Infeld action. The purpose of the present paper therefore is to construct this

[1] Note that although we refer to certain scalar fields of a non-BPS D-brane as tachyons, due to an orbifold projection, these modes are in fact massless.

action, to all orders in α', for a stack of stable non-BPS D-branes. The challenging part of this construction is to quantise open strings in a tachyonic background. For a generic point of the moduli space this is indeed a difficult problem [18, 19, 20]. However, at the point in moduli space where the tachyons become massless, the problem of quantising open strings in a constant tachyon background can be mapped to that of a non-abelian Wilson-line background by a suitable change of variables [21, 22]. In the new variables the condition for vanishing of the potential for the tachyon becomes simply a zero curvature condition which we will show to be equivalent to the non-abelian flat directions found in [17].

In the analysis of brane dynamics, an important role is played by the solitonic excitations on them. These usually have a spacetime interpretation as intersecting branes and provide information on some non-perturbative aspects of the worldvolume theory. These worldvolume solitons are also important in applications of D-branes to the study of Yang-Mills dynamics. Based on the worldvolume effective action constructed here we also include a discussion of the stable solitons on non-BPS D-branes.

Thus in this paper, we will be interested in the tree-level effective action for a stack of stable non-BPS branes. BI actions for unstable D-branes are discussed in [23, 24]. For the techincal reason mentioned above we will restrict our attention to the critical radius. We will also discuss some features of the stable soliton solutions on the non-BPS brane. Indeed it would appear that the more interesting solitons only arise at the critical radius. We leave the issue of string loop corrections and the effective potential to future work [25].

The plan for the rest of the paper is as follows. In the next section we briefly review the construction of a non-BPS D-brane in type-II string theory. In section three we then adapt the change of variables in [21, 22] to our present case. This then allows us to derive the complete low energy effective action in section four, although we will see that it is inherently non-abelian and so suffers the same problems that are encountered with the non-abelian Born-Infeld action for D-branes [6, 9]. Finally in section five we discuss some of the stable soliton states on a non-BPS brane and in particular their spacetime interpretation.

2. Review of stable non-BPS branes wrapped on $\mathbb{T}^4/\mathbb{Z}_2$.

We start by briefly recalling the key features non-BPS Dp-branes as presented in the review articles [16, 26]. These non-BPS Dp-brane ($\tilde{\mathrm{D}}p$-brane) have two types of open strings ending on it, labelled by the Chan-Paton indices I and σ_1. Contrary to BPS Dp-branes one has p even in type-IIB string theory and p odd in type-IIA string theory. The I-sector strings are precisely the same as for BPS D-branes and therefore their low energy modes form a maximally supersymmetric vector mulitplet in $(p+1)$ dimensions. The σ_1-sector strings come with the opposite projection under $(-1)^F$. Therefore, after the GSO projection, the level one modes of the NS-sector are projected out while the tachyonic ground state survives. Thus the lightest modes of the σ_1-sector consist of real tachyonic scalar with mass-squared $-1/2\alpha'$ and a massless sixteen-component fermion.

Clearly this system is unstable due to the tachyon. However, by wrapping the brane around an orbifold,[2] for example we will take $\mathbb{T}^4/\mathbb{Z}_2$ with coordinates $i = 6,\ldots,9$ and radii R_i, the tachyonic modes which are even under the orbifold action $g : x^i \leftrightarrow -x^i$ are projected out. Thus only the tachonic modes with odd momentum around the orbifold survive. Since a momemtum mode has mass-squared

$$m^2 = -\frac{1}{2\alpha'} + \sum_{i=6}^{9}\left(\frac{n_i}{R_i}\right)^2 , \tag{2.1}$$

we see that below a critical radius $R_c = \sqrt{2\alpha'}$ the surving states are non-tachyonic. In particular, with the conventions of [17], the four lightest tachyon scalars surviving the orbifold are given by (we set $\alpha' = 1$)

$$T^i = \frac{\chi^i(x)}{2i}\left(e^{i\vec{\omega}^i\cdot\vec{X}} - e^{-i\vec{\omega}^i\cdot\vec{X}}\right), \tag{2.2}$$

where $\vec{\omega}^i = (R_i)^{-1}\vec{e}^i$. In this way we obtain a stable non-BPS brane.

On the other hand, the effect of the orbifold on the bosons of the I-sector strings are the same as for a Dp-brane at an orbifold. Therefore, in the case of a D-sevenbrane wrapped on $\mathbb{T}^4/\mathbb{Z}_2$, which we will be most interested in here, the $N = 4$ vector multiplet is reduced to an $N = 2$ multiplet. This contains two scalars X^I, $I = 4,5$, that represent the fluctuations of the brane in the x^4 and x^5 directions, and a vector A_μ, $\mu = 0,1,2,3$. For the fermions the orbifold selects out one chirality under Γ^{6789} for the I-sector and the opposite for the σ_1-sector. This is the same fermionic content as four-dimensional $N = 4$ super-Yang-Mills. However even at the critical radius where the tachyonic modes are massless and the field content is precisely that of $N = 4$ super-Yang-Mills, the effective theory is not supersymmetric [17].

For the rest of this paper we will restrict our attention to oribifolds at the critical radius $R_i = \sqrt{2}$ where the lightest tachyon modes are massless. We will only use the Einstein summation convention for the full ten-dimensional indices $m,n = 0,1,2,\ldots,9$ and the four-dimensional indices $\mu,\nu = 0,1,2,3$. All other sums (e.g. those over the i,j and I,J indices) will be explicitly written.

3. Open strings in a tachyon background

In this section we adapt the change of variables introduced in [21, 16, 22] to the present situation. For simplicity we first consider the case of an abelian tachyon (i.e. for a non-BPS sevenbrane wrapped on $K3$) and the generators of the U(N) Lie algebra at the end.

[2] Note that here we are working in the T-dual picture to [17] where the branes were not wrapped around the orbifold.

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The objects of interest are the vertex operators for T^i in the (0) and (-1)-pictures. With the conventions of [17] but with $\alpha' = g_s = 1$ we then have

$$\begin{aligned} V_T^{(0)i} &= \frac{1}{\sqrt{2}}(\vec{\omega}^i\vec{\psi})\left(e^{i\vec{\omega}^i\cdot\vec{X}} + e^{-i\vec{\omega}^i\cdot\vec{X}}\right) \\ &= \frac{1}{2}\psi^i\left(e^{i\frac{1}{\sqrt{2}}X^i} + e^{-i\frac{1}{\sqrt{2}}X^i}\right). \end{aligned} \tag{3.1}$$

To continue we map the tachyon vertex operator (3.1) into a non-abelian Wilson line [21, 22]. Here we will only discuss in detail the (0)-picture but the relevant discussion for the (-1)-picture can be easily constructed by following the same change of variables but starting with

$$V_T^{(-1)i} = -\frac{i}{\sqrt{2}}e^{-\Phi}\left(e^{i\vec{\omega}^i\cdot\vec{X}} - e^{-i\vec{\omega}^i\cdot\vec{X}}\right), \tag{3.2}$$

where Φ is the bosonised superconformal ghost. We first express $V_T^{(0)i}$ in terms of the closed string fields $X^i_{R/L}, \psi^i_{R/L}$ with Neumann boundary condition

$$\begin{aligned} X^i_L &= X^i_R = \frac{1}{2}X^i\,, \\ \psi^i_L &= \psi^i_R = \psi^i\,, \end{aligned} \tag{3.3}$$

at both ends in the NS sector. In the R sector (3.3) applies at one end and

$$\psi^i_R = -\psi^i_L\,, \tag{3.4}$$

in the other. The vertex operator (3.1) can be written as

$$\frac{1}{2}\psi^i_R\left(e^{i\sqrt{2}X^i_R} + e^{-i\sqrt{2}X^i_R}\right). \tag{3.5}$$

At the critical orbifold ($R_i = R_c = \sqrt{2}$) we may fermionise $e^{i\sqrt{2}X^i_R}$ as

$$e^{i\sqrt{2}X^i_R} = \frac{1}{\sqrt{2}}(\xi^i_R + i\eta^i_R)\otimes\Gamma^i \qquad \text{and} \qquad e^{i\sqrt{2}X^i_L} = \frac{1}{\sqrt{2}}(\xi^i_L + i\eta^i_L)\otimes\Gamma^i\,, \tag{3.6}$$

where the cocycles Γ^i are introduced to restore the correct commutation relations with th world-sheet fermions [22, 27]. This can be achieved by taking for Γ^i the generators of the Spin(4)-Clifford algebra and attaching a Γ^5 to the world sheet fermions. We complete the transformation by rebosonising as

$$\frac{1}{\sqrt{2}}\left(\xi^i_{R/L} \pm i\psi^i_{R/L}\right) = e^{\pm i\sqrt{2}\tilde{X}^i_{R/L}}\otimes\tilde{\Gamma}^i\,, \tag{3.7}$$

and attaching a $\tilde{\Gamma}^5$ to $\eta^i_{R/L}$. Here the $\tilde{\Gamma}^i$ form another representation of the Spin(4)-Clifford algebra which commutes with the Γ^i representation. With these conventions the fermionic and bosonic currents are then related as

$$\eta^i_R\xi^i_R = i\sqrt{2}\partial X^i_R \qquad \text{and} \qquad \psi^i_R\xi^i_R = i\sqrt{2}\partial\tilde{X}^i_R\,. \tag{3.8}$$

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The boundary conditions for the different fields are determined as follows: from (3.1), (3.3) and (3.6) we have

$$\xi^i_L = \xi^i_R =: \xi^i \qquad \text{and} \qquad \eta^i_L = \eta^i_R =: \eta^i \,. \tag{3.9}$$

In the NS sector, where $\psi^i_L = \psi^i_R$ on both ends of the open string, (3.9) implies

$$\tilde{X}^i_L = \tilde{X}^i_R =: \frac{1}{2}\tilde{X}^i \,, \tag{3.10}$$

i.e. NN boundary conditions for $\tilde{X}^i$. In the R-sector, where $\psi^i_R(\pi) = -\psi^i_L(\pi)$, (3.9) implies in turn

$$\tilde{X}^i_L(\pi) = -\tilde{X}^i_R(\pi) \,, \tag{3.11}$$

i.e. ND boundary conditions for $\tilde{X}^i$. Using (3.8), (3.10) and (3.11) we may now write

$$\begin{aligned} V^{(0)i}_T &= i\partial_{||}\tilde{X}^i \otimes \sigma_1 \otimes \Gamma^5\Gamma^i \\ V^{(0)i}_T &= i\partial_{\perp}\tilde{X}^i \otimes \sigma_1 \otimes \Gamma^5\Gamma^i \,, \end{aligned} \tag{3.12}$$

in the NS and R sectors respectively. Finally the vertex operators $V^{(-1)i}$ in the (-1)-picture become simply

$$V^{(-1)i}_T = e^{-\Phi}\eta^i \otimes \sigma_1 \otimes \tilde{\Gamma}^5\Gamma^i \,. \tag{3.13}$$

To summarise, in the new variables $(\tilde{X}^i, \eta^i)$ the tachyon vev takes the form of a non-abelian Wilson line in the NS sector and of a shift in position in the R-sector respectively. In order to obtain the generalisation of (3.12) to non-abelian tachyon vevs all we need to do is to tensor (3.12) with an element of the u(N) Lie algebra. For example in the NS-sector we have

$$V^{(0)ia}_T = i\partial_{||}\tilde{X}^i \otimes \sigma_1 \otimes \Gamma^5\Gamma^i \otimes t^a \,. \tag{3.14}$$

In what follows we will omit the group indices so that, for example, χ^i is understood to mean $\chi^{ia} \otimes t^a$. The form (3.14) of the tachyon vertex operator suggests that switching on a vev for the χ^i's corresponds to an exact (in α') marginal deformation at the critical radius, provided $[V^i, V^j] = 0$, an assertion we shall verify explicitly in the next section. On the other hand, in the field theory, the condition $[V^i, V^j] = 0$ is equivalent to $\{\chi^i, \chi^j\} = 0$ using $\{\Gamma^i, \Gamma^j\} = 0$ for $i \neq j$. But this is precisely the conclusion reached in [17] based on the string S-matrix computation. The present result shows that these non-abelian flat directions persist to all orders in α'. We note also that it isn't necessary that all of the radii are critical. In order to perform the above change of variables we only need one radii to be critical which then leads to a flat direction. Of course we only obtain non-abelian flat directions if at least two of the radii are critical.

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4. Non-abelian BI action for stable non-BPS branes

Given that the tachyon vertex operators can be mapped into the form of non-abelian Wilson lines after an appropriate field redefinition, we can then compute the effective action within the existing formalism for Wilson lines [3]–[9]. The analysis is, maybe, most easily presented after T-dualising in the 4, 5-directions. In this case the positions of the branes are also given by Wilson lines. After we obtain the effective action for this case we can then T-dualise back to obtain the effective action for various non-BPS Dp-Branes. Now the positions are represented by the vertex operators

$$V^{(0)Ia} = i\partial_{||}X^I \otimes I \otimes I \otimes t^a \,. \tag{4.1}$$

Thus, we now consider $\tilde{\mathrm{D}}9$-branes wrapped around $\mathbb{T}^4/\mathbb{Z}_2 \times \mathbb{T}^2$, in the presence of a Wilson line $\mathcal{A}_m$, $m = 0, 1, 2, \ldots, 9$,

$$\mathcal{A}_m = \begin{cases} A^a_\mu \otimes I \otimes I \otimes t^a, & m = 0, \ldots, 3 \\ \phi^{Ia} \otimes I \otimes I \otimes t^a, & m = 4, 5 \\ \chi^{ia} \otimes \sigma^1 \otimes i\Gamma^5\Gamma^i \otimes t^a & m = 6, \ldots, 9\,, \end{cases} \tag{4.2}$$

where the hermitian matrices t^a represent the Lie algebra $\mathrm{u}(N)$. Here the additional factor of i in the definition of χ^i has been inserted to ensure that all the generators in (4.2) are hermitian.

As we have seen in the last section, at the critical radius, the (X^m, ψ^m) world-sheet CFT has an equivalent description in terms of the world-sheet fields[3]

$$X^\mu_{R/L}, X^I_{R/L}, \tilde{X}^i_{R/L} \qquad \text{and} \qquad \psi^\mu_{R/L}, \psi^I_{R/L}, \eta^i_{R/L}\,. \tag{4.3}$$

In these variables a tachyon vev is represented by a non-abelian Wilson line. Consequently the tree level effective action is obtained using the existing formalism [5]

$$\Gamma(\mathcal{A}_m) = \left\langle \mathrm{Tr}\, P \exp\left[i \int_0^{2\pi} ds \left(\dot{\mathcal{Y}}^m \mathcal{A}_m - \frac{1}{2}\lambda^m\lambda^n \mathcal{F}_{mn} \right)\right]\right\rangle, \tag{4.4}$$

with $\mathcal{Y}^m \in \{X^\mu, X^I, \tilde{X}^i\}$, $\lambda^m \in \{\psi^\mu, \psi^I, \eta^i\}$ and $\mathcal{F}_{mn} = \partial_m\mathcal{A}_n - \partial_n\mathcal{A}_m - ig[\mathcal{A}_m, \mathcal{A}_n]$. To continue we expand the integral in (4.4) as ($\mathcal{Y}^m = Y^m + \pi^m$, $\pi^m(2\pi) = \pi^m(0) = 0$)

$$\int ds \dot{\xi}^m A_m(Y + \pi) = \int ds \dot{\pi}^m \left[\frac{1}{2}\pi^n \mathcal{F}_{mn} + \frac{1}{3}\pi^n\pi^p D_p \mathcal{F}_{mn} + \cdots \right], \tag{4.5}$$

and perform a path integration over the fields π^m, λ^m. Thus the effective action for a non-BPS D-brane at the critical radius is of exactly the same form as for a non-abelian BPS D-brane but with the modified gauge connection (4.2). We note that, even for the case of a single non-BPS D-brane, where there are no t^a generators, the connection (4.2) is non-abelian.

[3]As only the NS sector contributes to the tree-level effective action we can ignore the modified boundary condition in the R sector in this section.

In general, for constant, but non-commuting Wilson lines the tree-level effective potential will receive corrections to all orders in α' and we were unable to find an analytic expression for that case. A simplification occurs if we neglect terms of the form $D_{(m}F_{n)p}$ [6]. In this case the 1-loop result (of the π^m, λ^m integration) is exact, leading to [6]

$$\Gamma(\mathcal{A}_m) = c_0 \mathrm{STr}\sqrt{\det(\delta_{mn} + 2\pi\mathcal{F}_{mn})}\,. \tag{4.6}$$

Here $\mathrm{STr}(M_1 \cdots M_n) = \frac{1}{n!}\sum_\sigma \mathrm{Tr}(M_{\sigma(1)} \cdots M_{\sigma(n)})$ is the symmeterised trace and σ is a permutation. Note that the determinate here is taken over m, n indices and the trace is over the Chan-Paton and cocycle factors in (4.2). Furthermore this expression is also valid when the Wilson lines are dynamical and therefore includes the kinetic terms for the bosonic fields.

While it might be thought that the approximation $D_{(m}\mathcal{F}_{n)p} \simeq 0$ effectively assumes the background to be abelian it has been argued [9] that the result may be valid in more general backgrounds. On the other hand, at the lowest non-trivial order (4.6) is exact and gives

$$\begin{aligned}
\Gamma &= c_0' \,\mathrm{Tr}\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\sum_I D_\mu\phi^I D^\mu\phi^I + \frac{1}{2}\sum_i D_\mu\chi^i D^\mu\chi^i - V\right), \\
V &= \frac{g^2}{4}\sum_{I,J}([\phi^I,\phi^J])^2 + \frac{g^2}{2}\sum_{I,j}([\phi^I,\chi^j])^2 - \frac{g^2}{4}\sum_{i\neq j}(\{\chi^i,\chi^j\})^2\,,
\end{aligned} \tag{4.7}$$

where c_0' is a constant and $D_\mu = \partial_\mu - ig[A_\mu,\ ]$. Here we have dropped a constant term in V and the trace is now only over the group indices a, b. This precisely reproduces the bosonic part of field theory approximation to the effective action found in [17] from string S-matrix calculations.

To continue we T-dualise the Born-Infeld action (4.6) in the $x^4, \ldots, x^9$- directions to obtain the BI-action for a stack of stable non-BPS $\tilde{\mathrm{D}}$3-branes on $\mathbb{T}^4/\mathbb{Z}_2(-1)^{F_L}$ [28, 29, 17]. In particular for $\phi^I = A_\mu = 0$ and constant χ^i

$$\Gamma[\chi^i] = c_0 \mathrm{STr}\sqrt{\det(\delta_{ij}I - 2\pi i g(\Gamma^{ij})\{\chi^i,\chi^j\})}\,. \tag{4.8}$$

The effective potential (4.8), which is the main result of this section, generalises the $O(\chi^4)$-potential found in [17].

For constant gauge potentials $\mathcal{F}_{AB} = -ig[\mathcal{A}_A, \mathcal{A}_B]$. This implies in particular that

$$\Gamma(\mathcal{A}_A) = \Gamma(0,0,0) \qquad \text{for} \qquad [\mathcal{A}_A, \mathcal{A}_B] = 0\,. \tag{4.9}$$

Thus the the condition for marginality at tree level but to all orders in α' is simply a zero-curvature constraint on the Wilson lines. In terms of the scalar fields (4.9) corresponds to

$$[\phi^I,\phi^J] = [\phi^I,\chi^j] = \{\chi^i,\chi^j\} = 0\,, \tag{4.10}$$

for all I, J, i, j with $i \neq j$. Note that the last condition is precisely the definition of marginality given above. In other words the tree level vacua of the field theory given by the zeros of the potential (4.7) and studied in [17] are exact to all orders in α'.

5. The BPS states of non-BPS branes

This section is devoted to some initial observations about the BPS states on non-BPS-branes viewed as solitons of the tree level effective action (4.6). Even though the low energy effective theory (4.6) has no supersymmetry one may still construct BPS bounds for the mass of soliton states in terms of conserved charges. While stability is not ensured by supersymmetry, we nevertheless expect them to enjoy at least classical stability since they are the lightest states carrying a given charge or satisfying a particular boundary condition.

Ideally we would like to derive BPS bounds for the full non-linear effective action. However, a closed expression for the non-abelian effective action is not known. For supersymmetric D-branes BPS bounds may be obtained from the supersymmetry algebra so that the detailed form of the non-linear effective action is not required. However, this option is not available to us here. Therefore it is not clear to us how to construct a BPS-type bound. Soliton solutions of the BPS non-abelian Born-Infeld action have been found in [31, 10, 32, 33, 34] and one might hope that these studies can be extended to the action (4.6). In this section we will therefore content ourselves by studying soliton solutions which saturate a BPS bound obtained from the lowest order term (4.7) in the effective action. We nevertheless expect that all the states we describe below will exist in the full non-linear theory.

Solitons on a D-brane generally have an interpretation as intersecting D-branes and the same ought to be true for non-BPS branes. Therefore before proceeding it is worthwhile to consider the stability of intersecting non-BPS branes. In the case of BPS D-branes it is well known that if the open strings which stretch between two D-branes have four coordinates with ND boundary conditions then some supersymmetry is preserved and the configuration is stable. For example this includes two Dp-branes intersecting over a $(p-2)$-brane or a D$(p-2)$-brane ending on a Dp-brane. For non-BPS branes we can easily see that the same configurations are stable since open strings with ND boundary conditions along four directions have a vanishing intercept and therefore no tachyons. Clearly tachyonic modes will not then be introduced by any GSO or orbifold projection.

The first class of BPS states that may be obtained correspond to setting $\chi^i = 0$. In this case the bosonic content of (4.7) is precisely the same as for $N = 2$ super-Yang-Mills theory. In particular there are three basic types of solitons and which we will now review and re-interpret.

In the case of a single stable non-BPS brane one may consider states with only the scalars ϕ^4 and ϕ^5 active. For string solitons that lie in the x^0, x^1 plane we write the energy density in the x^2, x^3 plane as

$$
\begin{aligned}
E &= \frac{1}{2}\int d^2x (\partial_2\phi^4)^2 + (\partial_2\phi^5)^2 + (\partial_3\phi^4)^2 + (\partial_3\phi^5)^2 \\
&= \frac{1}{2}\int d^2x (\partial_2\phi^4 - \partial_3\phi^5)^2 + (\partial_3\phi^4 + \partial_2\phi^5)^2 + T\,,
\end{aligned}
\tag{5.1}
$$

where $T = \partial_2(\phi^4\partial_3\phi^5) + \partial_3(-\phi^4\partial_2\phi^5)$ is a topological term. Thus the energy is minimised for a given boundary condition if $\partial_2\phi^4 = \partial_3\phi^5$ and $\partial_3\phi^4 = -\partial_2\phi^5$. These are just the Cauchy-Riemann equations for $\phi^4 + i\phi^5$ to be a holomorphic function of $x^2 + ix^3$. In the case of BPS D-branes this string soliton corresponds to the intersection of two D-threebranes [35]. Similarly, the soliton described here corresponds to the intersection of two non-BPS D-threebranes whose worldvolumes lie in the x^0, x^1, x^2, x^3 and x^0, x^1, x^4, x^5 planes respectively. Although the topological term generally gives a divergent energy contribution, there are solitons which are smooth at their core so that the effective action provides a reasonable approximation.

The other solitons that we will discuss correspond to states which are charged under some U(1) factor of a non-abelian gauge group U(N) of a stack of stable non-BPS branes at a critical orbifold. The central example being magnetic monopoles. Here we choose the gauge $A_0 = 0$ and consider static configurations with one ϕ^I active. We may therefore write

$$
\begin{aligned}
E &= \frac{1}{2}\int d^3x \frac{1}{2}|F_{\alpha\beta}|^2 + |D_\alpha\phi^I|^2 \\
&= \frac{1}{2}\int d^3x |B_\alpha - D_\alpha\phi^I|^2 + T\,,
\end{aligned}
\tag{5.2}
$$

where $\alpha, \beta = 1, 2, 3$, $B_\alpha = \frac{1}{2}\epsilon_{\alpha\beta\gamma}F^{\beta\gamma}$ is the magnetic field and $T = 2\partial_\alpha(B^\alpha\phi^I)$ is a topological term. Therefore, for a fixed charge given by the integral of the topological term, monopoles satisfying $B_\alpha = D_\alpha\phi^I$ are absolute minima of the effective action. For a BPS D-threebrane of type-IIB string theory the monopole corresponds to a D-string stretched between two of the D-threebranes along x^I [36, 37]. However, since the non-BPS D-threebranes occur in type-IIA string theory, these monopoles correspond to non-BPS D-strings stretched between two non-BPS D-threebranes.

There will also be electrically charged states, although these will not have finite energy to due divergences at their core. To obtain a bound for them we again consider static configurations with only one ϕ^I active but now with $A_a = 0$. In this case the energy can be written as (using the Gauss constraint $D_0\phi^I = 0$)

$$
\begin{aligned}
E &= \frac{1}{2}\int d^3x |\partial_a A_0|^2 + |\partial_a\phi^I|^2 \\
&= \frac{1}{2}\int d^3x |\partial_a(A_0 - \phi^I)|^2 + T\,,
\end{aligned}
\tag{5.3}
$$

where $T = 2\partial_a A_0 \partial^a \phi^I$. We therefore find stable electrically charged states if $A_0 = \phi^I$ and $\partial_a \partial^a \phi^I = 0$. These simply correspond to fundamental strings which end on the non-BPS D-threebrane an are infinitely extended along x^I [39, 40, 41].

Let us now consider solitons involving the scalar fields χ^i instead of the scalars ϕ^I. The string soliton mentioned above no longer exists due to the presence of the $(\chi^i \chi^j)^2$ term in the potential of (4.7). However the charged solutions certainly do exist since if only one χ^i is active then the potential term vanishes just as it did above for ϕ^I. Thus there are BPS saturated monopole states with $B_a = D_a \chi^i$ and electric states with $A_0 = \chi^i$ too.

Note that turning on the tachyon vev $\chi^i = 1$ corresponds to deforming the non-BPS D-threebrane into a D-fourbrane/anti-D-fourbrane pair wrapped along x^i with half a unit of Wilson line [16]. Therefore a monopole soliton involving χ^i can be pictured as two non-BPS D-threebranes at the core which at infinity fatten out into two D-fourbrane/anti-D-fourbrane pairs wrapped around x^i, with a non-abelian Wilson line turned on. These states again correspond to non-BPS D-strings but this time stretched along an orbifold direction x^i. Similarly the most natural interpretation for the electric states is that they correspond to a fundamental string stretched along x^i and ending on the non-BPS D-threebranes.

Next we consider what happens if we move away from the critical radius. In this case the mass term $\frac{1}{2} m_i^2 (\chi^i)^2$ appears in the action (4.7) [17]. The solutions we just described involving χ^i then no longer exist. In the case of the monopole this can be understood because a non-BPS D-threebrane is stable and hence the m_i^2 are positive only if the orbifold radii are larger than the critical radius. However, a non-BPS D-string stretched along an orbifold direction is stable only if the radius is less than critical. Therefore the two can only be simultaneously stable precisely at the critical radius. For the electrically charged states, however, it is not clear to us what the origin of the instability is.

Finally we note that Yang-Mills theories possesses other solitons including the so-called 1/4-BPS states with two scalars active [38]. Clearly these solitons also appear in the action (4.7). In general though these solitons won't exist if both scalars are taken to be tachyon modes. In addition to the states we described above there will also be "mixed" solitons where some of the ϕ^I and only one of the χ^i scalars are active since in these cases the effective action is same as $N = 4$ super-Yang-Mills. We would also like to highlight the possibility that the effective action (4.7) admits new types of solitons associated with the non-abelian flat directions in the tachyon potential.

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